

# MODELING OF NONLINEAR ACTIVE AND PASSIVE DEVICES IN THREE-DIMENSIONAL TLM NETWORKS

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## ABSTRACT

We propose an extension for the TLM-SC node to analyze three-dimensional problems including nonlinear active and passive devices. The nonlinear behavior of the lumped circuit is decoupled from the impulse scattering at the nodes, yielding a general and systematic methodology to embed arbitrary nonlinear devices into the TLM mesh. Two canonical examples are given to demonstrate the versatility of the method. These include a Tunnel diode oscillator with DC biasing network and the modeling of a pn-junction diode.

## INTRODUCTION

In recent years, two numerical methods have proved to be very efficient and accurate in solving electromagnetic problems in the time-domain: the Transmission Line Matrix method (TLM) [1] and the Finite Difference Time Domain method (FDTD) [2]. The extension of these two techniques to include lumped elements [3-4], and in particular nonlinear and active devices ([5-12]), in distributed systems is a natural need to tackle more complex electromagnetic problems.

The TLM method happens to be particularly suitable for interfacing lumped circuits to distributed structures: the electromagnetic fields are directly related to the voltages and currents propagating in the TLM transmission lines, and it is straightforward to interpret the connection between the distributed electromagnetic problem and the lumped devices in terms of circuit theory. Moreover, unlike FDTD, the TLM method is very stable at low frequencies, particularly in the presence of absorbing boundary conditions [13].

In this paper, we propose an extension for the TLM-SC node [14], to analyze three-dimensional hybrid systems.

The incorporation of lumped circuits, including active nonlinear devices and biasing networks, will be performed by connecting the lumped elements to the nodes of a TLM network via a set of transmission lines of length  $\Delta l/2$ ; the entire region will be modeled as a single element: this will prevent the generation of instabilities if active devices are present in the network. The generality of the approach is obtained by decoupling the equation describing the behavior of the device from the scattering procedure at the TLM nodes.

The accuracy of the proposed technique will be validated by analyzing canonical problems, showing always a very good agreement between the simulations and the expected results.

## MODELING OF LUMPED ELEMENT CIRCUITS WITH THE TLM-SC-NODE

In the following we describe the technique to introduce lumped elements in a set of TLM-SC nodes. To incorporate the circuit into the TLM mesh, we will consider an equivalent region extending over the device volume, but unlike the two-dimensional case [5], we will not assume that the behavior of the device in each cell is dependent only on the local field in that cell [10] (Fig. 1). This is particularly important in the case of nonlinear active elements, such as Tunnel diodes, because the connection of the cells in the direction of the feeding voltage will have the effect of a series of diodes, which is DC-unstable. A second advantage is that when modeling nonlinear devices such as pn-junction diodes or bipolar transistors, we will need to solve only a single nonlinear equation at each iteration.

To connect the device to the TLM mesh we add  $n$  series-connected capacitive stubs in the direction of the feeding voltage (in this case,  $y$  direction) (Fig. 2); for simplicity, we also assume that the circuit occupies only one

TLM cell in the  $x$ - $z$  plane.

By choosing appropriately the value of the normalized stub admittance ( $\hat{Y}_y = 4$ ) [5], the incident and reflected voltages at the transmission line connecting the device to the TLM node can be decoupled.

The inclusion of the circuit into the TLM mesh is performed by expressing the equation relating the voltage across the element,  $v_y$ , and current flowing into it,  $i$ , as a function of the voltages traveling in the  $n$  TLM stubs:

$$v = v_y = \sum_{m=1}^n v_m = \sum_{m=1}^n (v_m^i + v_m^r) \quad (1)$$

$$i = i_m = Y_r (v_m^r - v_m^i) \quad m = 1, \dots, n \quad (2)$$

Note that in the second equation we have assumed that all the stubs have the same characteristic admittance,  $Y_r = \hat{Y}_y Y_0$ .

From the system of equations (2) we can derive all the incident voltages  $v_m^i$  as a function of only one of them, for example  $v_1^i$ ; hence, (1) and (2) can be rearranged to give:

$$v_m^i = v_m^r - v_1^r + v_1^i \quad m = 2, \dots, n \quad (3)$$

$$v = n v_1^i + (2 - n) v_1^r + 2 \sum_{m=2}^n v_m^r \quad (4)$$

Substituting the expressions for the current (2) and for the voltage (4) in the equation describing the lumped circuit and applying an appropriate discretization scheme (we have used a central difference scheme), we obtain a recursive formulation for the new incident voltages traveling in the stubs. For first-order circuits this can be written as:

$${}_k V_{s1}^i = F \left[ {}_k V_{s1}^r, {}_{k-1} V_{s1}^i, {}_{k-1} V_{s1}^r, \sum_{m=2}^n {}_k V_{sm}^r, \sum_{m=2}^n {}_{k-1} V_{sm}^r \right] \quad (5)$$

$${}_k V_{sm}^i = {}_k V_{sm}^r - {}_k V_{s1}^r + {}_k V_{s1}^i \quad m = 2, \dots, n \quad (6)$$

where  $k$  represents the discretized time-step  $k\Delta t$ .

This formula must be particularized for each kind of lumped circuit that we want to include into the TLM scheme.

## RESULTS

We have simulated an air-filled stripline oscillator with its DC biasing circuit (Fig. 3). The two diodes were assumed to be equal, and to have as biasing point  $V_0 = 1$  V and  $I_0 = 3.5$  mA. The typical I-V characteristic of the diode has been approximated with a nonlinear third-order polynomial:

$$i_1(v_1) = (19.5 - 24.0v_1 + 8v_1^2) v_1 \quad (mA) \quad (7)$$

We also assumed that the active devices comprise a series resistance of value  $R_s = 4 \Omega$ , and a shunt capacitance  $C_p = 1$  pF. The source model represented a DC voltage supply of value  $V_s = 1.4$  V, with an internal resistance  $R_{int} = 55.143 \Omega$ .

The voltage across one of the diodes is shown in Fig. 4. When the oscillations reach the steady-state, the measured mean values of the voltage across the diode and of the current flowing into it, are respectively 0.993 V and 3.46 mA, showing a good agreement with the imposed biasing point. The difference between the SPICE and the TLM simulations during the transient period is due to the adaptive time-step used by SPICE. The frequency response in Fig. 5 has been evaluated after the steady-state condition has been reached ( $t > 2.915$  ns). By comparing the TLM results with the SPICE simulation, a slight difference in the frequency of oscillation is observed; this is due to the presence of a distributed capacitance between the diode and the magnetic wall, not considered in the SPICE model.

As a second example we have modeled the static behavior of a pn-junction diode [15], having a saturation current of  $10^{-14}$  A and placed at one extremity of a stripline. The excitation was a matched resistive voltage source, providing a 10-volt, 200MHz sinusoid. The characteristics of the stripline for this experiment were:  $l = 20$  mm,  $h = 1$  mm and  $w = 4$  mm.

The TLM analysis of the given structure has been compared with the results obtained using SPICE (Fig. 6), showing a very good agreement, both for the voltage and current behavior. The maximum voltage and current deviations between the two approaches, are of 83.4 mV and 0.59 mA, respectively.

## CONCLUSIONS

We have proposed an extension of the TLM method to analyze three-dimensional hybrid problems, consisting of distributed and lumped components. The inclusion of lumped devices has been performed by modeling the entire circuit region as a single element, to prevent the generation of instabilities in the network and by decoupling the equation describing the circuit from the TLM scattering procedure. The new technique has been specialized for several kinds of circuits, and has proven to be capable of correctly describing the transient response of nonlinear devices, including the DC behavior of the active elements.

The accuracy of the proposed method has been vali-

dated by comparing the results of our approach with the analytical solutions or with SPICE simulations.

Future work includes the introduction of more complex and detailed models of active elements into the TLM mesh; in particular, an immediate extension will be the inclusion of large signal equivalent circuits for transistors and parasitic effects.

#### ACKNOWLEDGEMENTS

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#### FIGURES

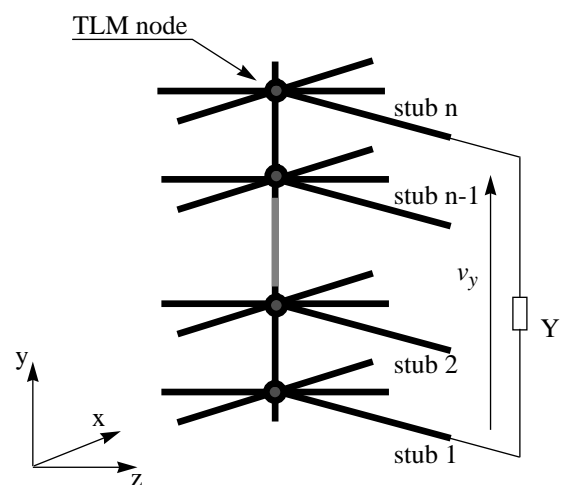


Fig. 1 Connection of the nonlinear device to the three-dimensional TLM mesh: the circuit region is modeled as a single element

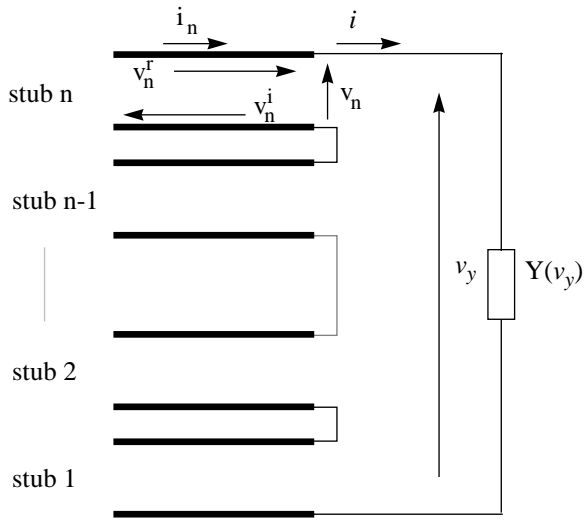


Fig. 2 Connection of a nonlinear device, modeled by an equivalent circuit, to the TLM mesh

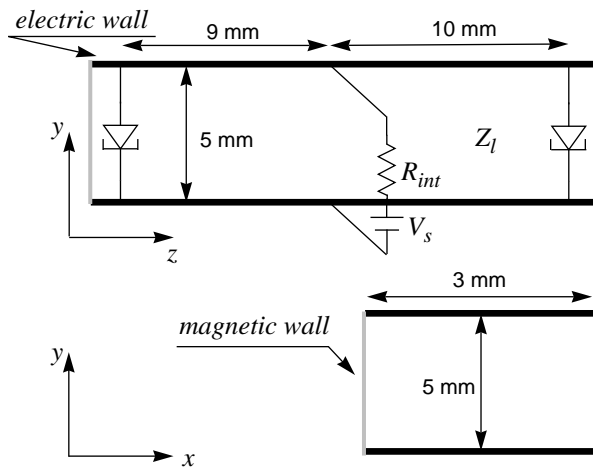


Fig. 3 Stripline oscillator with DC biasing circuit

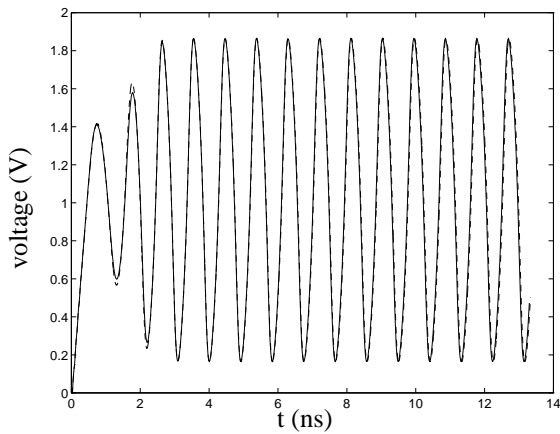


Fig. 4 Time response for one of the diodes in the stripline oscillator shown in Fig. 3 (Solid line: SPICE, dashed line: TLM)

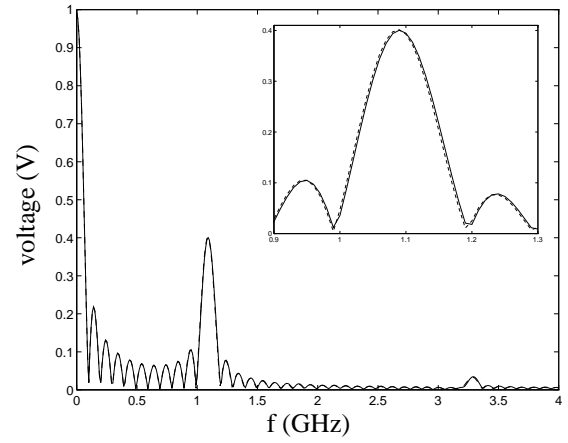


Fig. 5 Frequency response for one of the diodes in the stripline oscillator shown in Fig. 3 (Solid line: SPICE, dashed line: TLM)

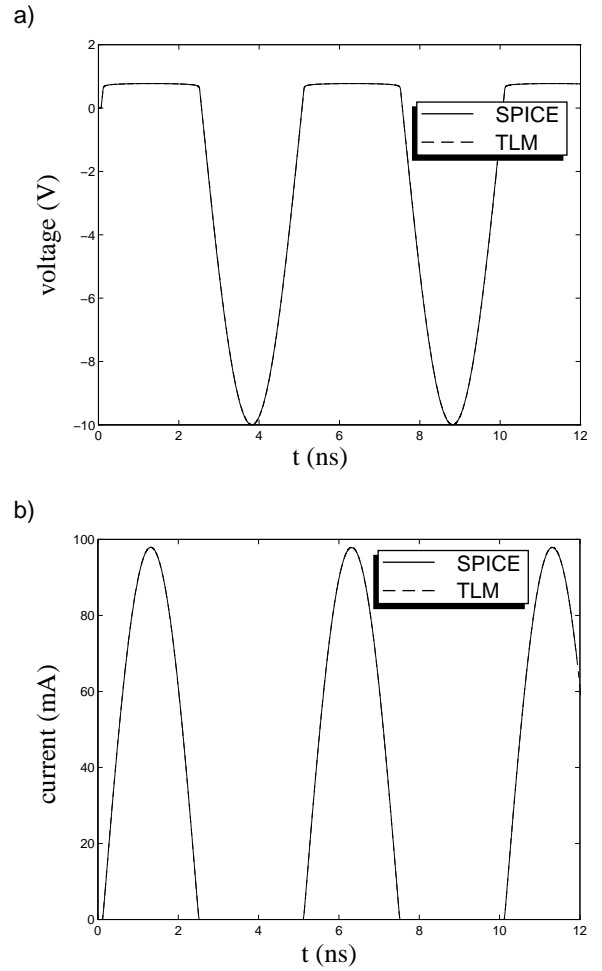


Fig. 6 Voltage (a) and current (b) behavior for the pn-junction diode: comparison between the TLM simulation and SPICE results ( $\Delta V_{\max} = 83.4$  mV,  $\Delta I_{\max} = 0.59$  mA)